

12

Static Equilibrium and Elasticity

CHAPTER OUTLINE

- 12.1 The Conditions for Equilibrium
- 12.2 More on the Center of Gravity
- 12.3 Examples of Rigid Objects in Static Equilibrium
- 12.4 Elastic Properties of Solids

ANSWERS TO QUESTIONS

- Q12.1** When you bend over, your center of gravity shifts forward. Once your CG is no longer over your feet, gravity contributes to a nonzero net torque on your body and you begin to rotate.
- Q12.2** Yes, it can. Consider an object on a spring oscillating back and forth. In the center of the motion both the sum of the torques and the sum of the forces acting on the object are (separately) zero. Again, a meteoroid flying freely through interstellar space feels essentially no forces and keeps moving with constant velocity.
- Q12.3** No—one condition for equilibrium is that $\sum \mathbf{F} = 0$. For this to be true with only a single force acting on an object, that force would have to be of zero magnitude; so really no forces act on that object.
- Q12.4** (a) Consider pushing up with one hand on one side of a steering wheel and pulling down equally hard with the other hand on the other side. A pair of equal-magnitude oppositely-directed forces applied at different points is called a couple.
- (b) An object in free fall has a non-zero net force acting on it, but a net torque of zero about its center of mass.
- Q12.5** No. If the torques are all in the same direction, then the net torque cannot be zero.
- Q12.6** (a) Yes, provided that its angular momentum is constant.
- (b) Yes, provided that its linear momentum is constant.
- Q12.7** A V-shaped boomerang, a barstool, an empty coffee cup, a satellite dish, and a curving plastic slide at the edge of a swimming pool each have a center of mass that is not within the bulk of the object.
- Q12.8** Suspend the plywood from the nail, and hang the plumb bob from the nail. Trace on the plywood along the string of the plumb bob. Now suspend the plywood with the nail through a different point on the plywood, not along the first line you drew. Again hang the plumb bob from the nail and trace along the string. The center of gravity is located halfway through the thickness of the plywood under the intersection of the two lines you drew.

- Q12.9** The center of gravity must be directly over the point where the chair leg contacts the floor. That way, no torque is applied to the chair by gravity. The equilibrium is unstable.
- Q12.10** She can be correct. If the dog stands on a relatively thick scale, the dog's legs on the ground might support more of its weight than its legs on the scale. She can check for and if necessary correct for this error by having the dog stand like a bridge with two legs on the scale and two on a book of equal thickness—a physics textbook is a good choice.
- Q12.11** If their base areas are equal, the tall crate will topple first. Its center of gravity is higher off the incline than that of the shorter crate. The taller crate can be rotated only through a smaller angle before its center of gravity is no longer over its base.
- Q12.12** The free body diagram demonstrates that it is necessary to have friction on the ground to counterbalance the normal force of the wall and to keep the base of the ladder from sliding. Interestingly enough, if there is friction on the floor *and* on the wall, it is not possible to determine whether the ladder will slip from the equilibrium conditions alone.

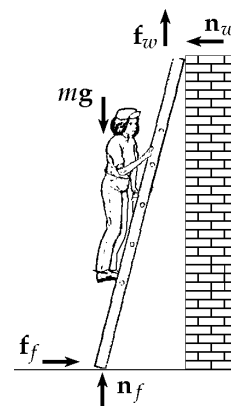


FIG. Q12.12

- Q12.13** When you lift a load with your back, your back muscles must supply the torque not only to rotate your upper body to a vertical position, but also to lift the load. Since the distance from the pivot—your hips—to the load—essentially your shoulders—is great, the force required to supply the lifting torque is very large. When lifting from your knees, your back muscles need only keep your back straight. The force required to do that is much smaller than when lifting with your back, as the torque required is small, because the moment arm of the load is small—the line of action of the load passes close to your hips. When you lift from your knees, your much stronger leg and hip muscles do the work.
- Q12.14** Shear deformation.
- Q12.15** The vertical columns experience simple compression due to gravity acting upon their mass. The horizontal slabs, however, suffer significant shear stress due to gravity. The bottom surface of a sagging lintel is under tension. Stone is much stronger under compression than under tension, so horizontal slabs are more likely to fail.

SOLUTIONS TO PROBLEMS

Section 12.1 The Conditions for Equilibrium

P12.1 To hold the bat in equilibrium, the player must exert both a force and a torque on the bat to make

$$\sum F_x = \sum F_y = 0 \text{ and } \sum \tau = 0$$

$\sum F_y = 0 \Rightarrow F - 10.0 \text{ N} = 0$, or the player must exert a net upward force of $F = \boxed{10.0 \text{ N}}$

To satisfy the second condition of equilibrium, the player must exert an applied torque τ_a to make

$\sum \tau = \tau_a - (0.600 \text{ m})(10.0 \text{ N}) = 0$. Thus, the required torque is

$$\tau_a = +6.00 \text{ N} \cdot \text{m} \text{ or } \boxed{6.00 \text{ N} \cdot \text{m} \text{ counterclockwise}}$$

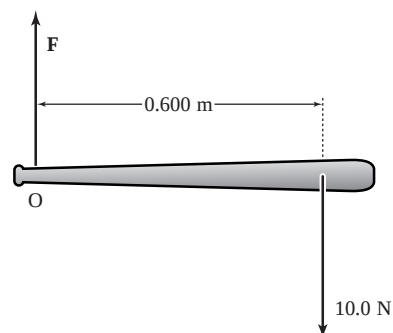


FIG. P12.1

P12.2 Use distances, angles, and forces as shown. The conditions of equilibrium are:

$$\sum F_y = 0 \Rightarrow \boxed{F_y + R_y - F_g = 0}$$

$$\sum F_x = 0 \Rightarrow \boxed{F_x - R_x = 0}$$

$$\sum \tau = 0 \Rightarrow \boxed{F_y \ell \cos \theta - F_g \left(\frac{\ell}{2} \right) \cos \theta - F_x \ell \sin \theta = 0}$$

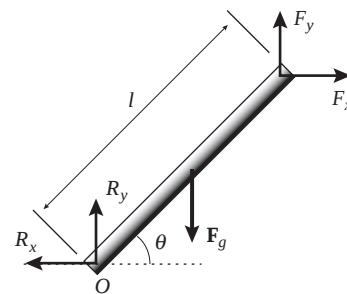


FIG. P12.2

P12.3 Take torques about P .

$$\sum \tau_p = -n_0 \left[\frac{\ell}{2} + d \right] + m_1 g \left[\frac{\ell}{2} + d \right] + m_b g d - m_2 g x = 0$$

We want to find x for which $n_0 = 0$.

$$x = \frac{(m_1 g + m_b g)d + m_1 g \frac{\ell}{2}}{m_2 g} = \boxed{\frac{(m_1 + m_b)d + m_1 \frac{\ell}{2}}{m_2}}$$

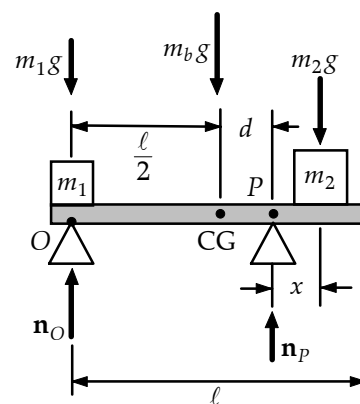


FIG. P12.3

Section 12.2 More on the Center of Gravity

P12.4 The hole we can count as negative mass

$$x_{\text{CG}} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

Call σ the mass of each unit of pizza area.

$$x_{\text{CG}} = \frac{\sigma \pi R^2 0 - \sigma \pi \left(\frac{R}{2}\right)^2 \left(-\frac{R}{2}\right)}{\sigma \pi R^2 - \sigma \pi \left(\frac{R}{2}\right)^2}$$

$$x_{\text{CG}} = \frac{\frac{R}{8}}{\frac{3}{4}} = \boxed{\frac{R}{6}}$$

P12.5 The coordinates of the center of gravity of piece 1 are

$$x_1 = 2.00 \text{ cm} \text{ and } y_1 = 9.00 \text{ cm}.$$

The coordinates for piece 2 are

$$x_2 = 8.00 \text{ cm} \text{ and } y_2 = 2.00 \text{ cm}.$$

The area of each piece is

$$A_1 = 72.0 \text{ cm}^2 \text{ and } A_2 = 32.0 \text{ cm}^2.$$

And the mass of each piece is proportional to the area. Thus,

$$x_{\text{CG}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(72.0 \text{ cm}^2)(2.00 \text{ cm}) + (32.0 \text{ cm}^2)(8.00 \text{ cm})}{72.0 \text{ cm}^2 + 32.0 \text{ cm}^2} = \boxed{3.85 \text{ cm}}$$

and

$$y_{\text{CG}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(72.0 \text{ cm}^2)(9.00 \text{ cm}) + (32.0 \text{ cm}^2)(2.00 \text{ cm})}{104 \text{ cm}^2} = \boxed{6.85 \text{ cm}}.$$

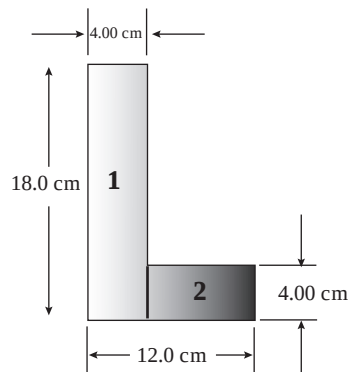


FIG. P12.5

- P12.6** Let σ represent the mass-per-face area. A vertical strip at position x , with width dx and height $\frac{(x-3.00)^2}{9}$ has mass

$$dm = \frac{\sigma(x-3.00)^2 dx}{9}.$$

The total mass is

$$M = \int dm = \int_{x=0}^{3.00} \frac{\sigma(x-3)^2 dx}{9}$$

$$M = \left(\frac{\sigma}{9}\right) \int_0^{3.00} (x^2 - 6x + 9) dx$$

$$M = \left(\frac{\sigma}{9}\right) \left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x \right]_0^{3.00} = \sigma$$

The x -coordinate of the center of gravity is

$$x_{CG} = \frac{\int x dm}{M} = \frac{1}{9\sigma} \int_0^{3.00} \sigma x(x-3)^2 dx = \frac{\sigma}{9\sigma} \int_0^{3.00} (x^3 - 6x^2 + 9x) dx = \frac{1}{9} \left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{9x^2}{2} \right]_0^{3.00} = \frac{6.75 \text{ m}}{9.00} = \boxed{0.750 \text{ m}}$$

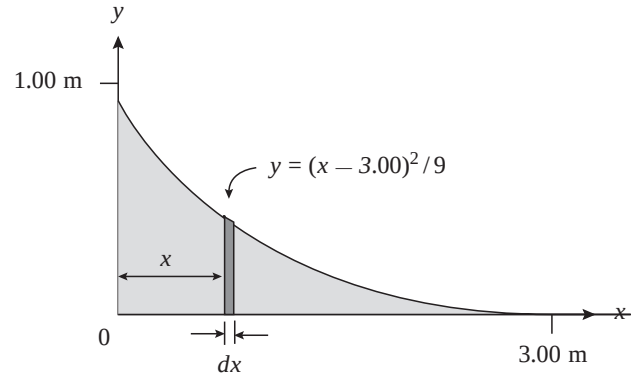


FIG. P12.6

- P12.7** Let the fourth mass (8.00 kg) be placed at (x, y) , then

$$x_{CG} = 0 = \frac{(3.00)(4.00) + m_4(x)}{12.0 + m_4}$$

$$x = -\frac{12.0}{8.00} = \boxed{-1.50 \text{ m}}$$

Similarly, $y_{CG} = 0 = \frac{(3.00)(4.00) + 8.00(y)}{12.0 + 8.00}$

$$y = \boxed{-1.50 \text{ m}}$$

- P12.8** In a uniform gravitational field, the center of mass and center of gravity of an object coincide. Thus, the center of gravity of the triangle is located at $x = 6.67 \text{ m}$, $y = 2.33 \text{ m}$ (see the Example on the center of mass of a triangle in Chapter 9).

The coordinates of the center of gravity of the three-object system are then:

$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(6.00 \text{ kg})(5.50 \text{ m}) + (3.00 \text{ kg})(6.67 \text{ m}) + (5.00 \text{ kg})(-3.50 \text{ m})}{(6.00 + 3.00 + 5.00) \text{ kg}}$$

$$x_{CG} = \frac{35.5 \text{ kg} \cdot \text{m}}{14.0 \text{ kg}} = \boxed{2.54 \text{ m}} \text{ and}$$

$$y_{CG} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(6.00 \text{ kg})(7.00 \text{ m}) + (3.00 \text{ kg})(2.33 \text{ m}) + (5.00 \text{ kg})(+3.50 \text{ m})}{14.0 \text{ kg}}$$

$$y_{CG} = \frac{66.5 \text{ kg} \cdot \text{m}}{14.0 \text{ kg}} = \boxed{4.75 \text{ m}}$$

Section 12.3 Examples of Rigid Objects in Static Equilibrium

P12.9 $\sum \tau = 0 = mg(3r) - Tr$
 $2T - Mg \sin 45.0^\circ = 0$
 $T = \frac{Mg \sin 45.0^\circ}{2} = \frac{1\,500 \text{ kg}(g) \sin 45.0^\circ}{2}$
 $= (530)(9.80) \text{ N}$
 $m = \frac{T}{3g} = \frac{530g}{3g} = \boxed{177 \text{ kg}}$

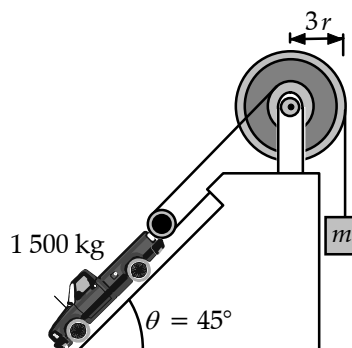


FIG. P12.9

***P12.10** (a) For rotational equilibrium of the lowest rod about its point of support, $\sum \tau = 0$.

$$+12.0 \text{ g} g 3 \text{ cm} - m_1 g 4 \text{ cm} = 0 \quad \boxed{m_1 = 9.00 \text{ g}}$$

(b) For the middle rod,

$$+m_2 2 \text{ cm} - (12.0 \text{ g} + 9.0 \text{ g}) 5 \text{ cm} = 0 \quad \boxed{m_2 = 52.5 \text{ g}}$$

(c) For the top rod,

$$(52.5 \text{ g} + 12.0 \text{ g} + 9.0 \text{ g}) 4 \text{ cm} - m_3 6 \text{ cm} = 0 \quad \boxed{m_3 = 49.0 \text{ g}}$$

P12.11 $F_g \rightarrow$ standard weight
 $F'_g \rightarrow$ weight of goods sold
 $F_g(0.240) = F'_g(0.260)$

$$F_g = F'_g \left(\frac{13}{12} \right)$$

$$\left(\frac{F_g - F'_g}{F'_g} \right) 100 = \left(\frac{13}{12} - 1 \right) \times 100 = \boxed{8.33\%}$$

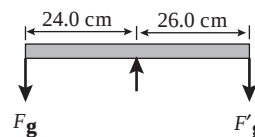


FIG. P12.11

***P12.12** (a) Consider the torques about an axis perpendicular to the page and through the left end of the horizontal beam.

$$\sum \tau = +(T \sin 30.0^\circ)d - (196 \text{ N})d = 0,$$

$$\text{giving } T = \boxed{392 \text{ N}}.$$

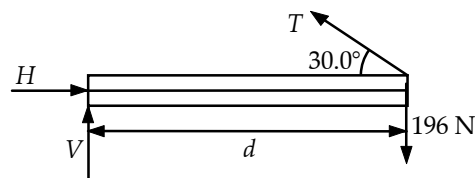


FIG. P12.12

(b) From $\sum F_x = 0$, $H - T \cos 30.0^\circ = 0$, or $H = (392 \text{ N}) \cos 30.0^\circ = \boxed{339 \text{ N to the right}}$.

From $\sum F_y = 0$, $V + T \sin 30.0^\circ - 200 \text{ N} = 0$, or $V = 196 \text{ N} - (392 \text{ N}) \sin 30.0^\circ = \boxed{0}$.

P12.13 (a) $\sum F_x = f - n_w = 0$

$$\sum F_y = n_g - 800 \text{ N} - 500 \text{ N} = 0$$

Taking torques about an axis at the foot of the ladder,

$$(800 \text{ N})(4.00 \text{ m})\sin 30.0^\circ + (500 \text{ N})(7.50 \text{ m})\sin 30.0^\circ$$

$$-n_w(15.0 \text{ cm})\cos 30.0^\circ = 0$$

Solving the torque equation,

$$n_w = \frac{[(4.00 \text{ m})(800 \text{ N}) + (7.50 \text{ m})(500 \text{ N})]\tan 30.0^\circ}{15.0 \text{ m}} = 268 \text{ N}.$$

Next substitute this value into the F_x equation to find

$$f = n_w = \boxed{268 \text{ N}} \text{ in the positive } x \text{ direction.}$$

Solving the equation $\sum F_y = 0$,

$$n_g = \boxed{1300 \text{ N}} \text{ in the positive } y \text{ direction.}$$

(b) In this case, the torque equation $\sum \tau_A = 0$ gives:

$$(9.00 \text{ m})(800 \text{ N})\sin 30.0^\circ + (7.50 \text{ m})(500 \text{ N})\sin 30.0^\circ - (15.0 \text{ m})(n_w)\sin 60.0^\circ = 0$$

or $n_w = 421 \text{ N}.$

Since $f = n_w = 421 \text{ N}$ and $f = f_{\max} = \mu n_g$, we find

$$\mu = \frac{f_{\max}}{n_g} = \frac{421 \text{ N}}{1300 \text{ N}} = \boxed{0.324}.$$

P12.14 (a) $\sum F_x = f - n_w = 0$ (1)

$$\sum F_y = n_g - m_1 g - m_2 g = 0$$
 (2)

$$\sum \tau_A = -m_1 g \left(\frac{L}{2} \right) \cos \theta - m_2 g x \cos \theta + n_w L \sin \theta = 0$$

From the torque equation,

$$n_w = \left[\frac{1}{2} m_1 g + \left(\frac{x}{L} \right) m_2 g \right] \cot \theta$$

Then, from equation (1): $f = n_w = \left[\frac{1}{2} m_1 g + \left(\frac{x}{L} \right) m_2 g \right] \cot \theta$

and from equation (2): $n_g = \boxed{(m_1 + m_2)g}$

(b) If the ladder is on the verge of slipping when $x = d$,

then
$$\mu = \frac{f|_{x=d}}{n_g} = \frac{\left(\frac{m_1}{2} + \frac{m_2 d}{L} \right) \cot \theta}{m_1 + m_2}.$$

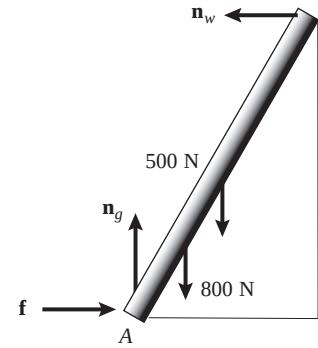


FIG. P12.13

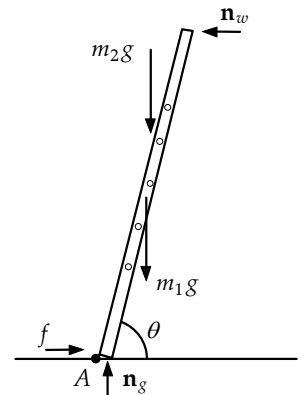


FIG. P12.14

P12.15 (a) Taking moments about P ,

$$(R \sin 30.0^\circ)(0) + (R \cos 30.0^\circ)(5.00 \text{ cm}) - (150 \text{ N})(30.0 \text{ cm}) = 0$$

$$R = 1039.2 \text{ N} = 1.04 \text{ kN}$$

The force exerted by the hammer on the nail is equal in magnitude and opposite in direction:

$$1.04 \text{ kN at } 60^\circ \text{ upward and to the right.}$$

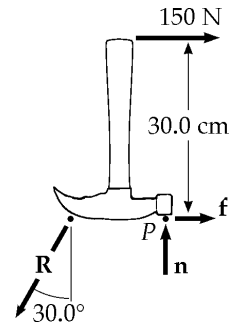


FIG. P12.15

(b) $f = R \sin 30.0^\circ - 150 \text{ N} = 370 \text{ N}$

$$n = R \cos 30.0^\circ = 900 \text{ N}$$

$$\mathbf{F}_{\text{surface}} = (370 \text{ N})\hat{\mathbf{i}} + (900 \text{ N})\hat{\mathbf{j}}$$

P12.16 See the free-body diagram at the right.

When the plank is on the verge of tipping about point P , the normal force n_1 goes to zero. Then, summing torques about point P gives

$$\sum \tau_p = -mgd + Mgx = 0 \quad \text{or} \quad x = \left(\frac{m}{M}\right)d.$$

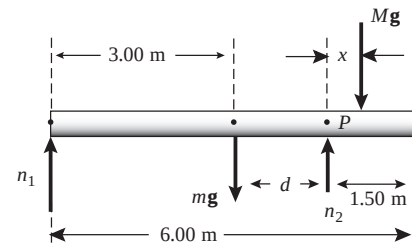


FIG. P12.16

From the dimensions given on the free-body diagram, observe that $d = 1.50 \text{ m}$. Thus, when the plank is about to tip,

$$x = \left(\frac{30.0 \text{ kg}}{70.0 \text{ kg}}\right)(1.50 \text{ m}) = 0.643 \text{ m}.$$

P12.17 Torque about the front wheel is zero.

$$0 = (1.20 \text{ m})(mg) - (3.00 \text{ m})(2F_r)$$

Thus, the force at each rear wheel is

$$F_r = 0.200mg = 2.94 \text{ kN}.$$

The force at each front wheel is then

$$F_f = \frac{mg - 2F_r}{2} = 4.41 \text{ kN}.$$

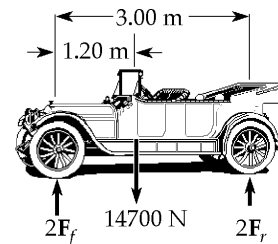


FIG. P12.17

P12.18 $\sum F_x = F_b - F_t + 5.50 \text{ N} = 0$ (1)

$$\sum F_y = n - mg = 0$$

Summing torques about point O,

$$\sum \tau_O = F_t(1.50 \text{ m}) - (5.50 \text{ m})(10.0 \text{ m}) = 0$$

which yields $F_t = \boxed{36.7 \text{ N to the left}}$

Then, from Equation (1),

$$F_b = 36.7 \text{ N} - 5.50 \text{ N} = \boxed{31.2 \text{ N to the right}}$$

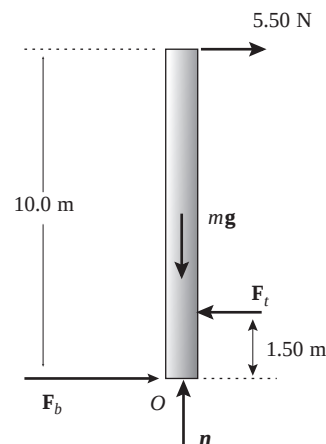


FIG. P12.18

P12.19 (a) $T_e \sin 42.0^\circ = 20.0 \text{ N}$ $\boxed{T_e = 29.9 \text{ N}}$

(b) $T_e \cos 42.0^\circ = T_m$ $\boxed{T_m = 22.2 \text{ N}}$

P12.20 Relative to the hinge end of the bridge, the cable is attached horizontally out a distance $x = (5.00 \text{ m}) \cos 20.0^\circ = 4.70 \text{ m}$ and vertically down a distance $y = (5.00 \text{ m}) \sin 20.0^\circ = 1.71 \text{ m}$. The cable then makes the following angle with the horizontal:

$$\theta = \tan^{-1} \left[\frac{(12.0 + 1.71) \text{ m}}{4.70 \text{ m}} \right] = 71.1^\circ.$$

(a) Take torques about the hinge end of the bridge:

$$\begin{aligned} R_x(0) + R_y(0) - 19.6 \text{ kN}(4.00 \text{ m}) \cos 20.0^\circ \\ - T \cos 71.1^\circ(1.71 \text{ m}) + T \sin 71.1^\circ(4.70 \text{ m}) \\ - 9.80 \text{ kN}(7.00 \text{ m}) \cos 20.0^\circ = 0 \end{aligned}$$

which yields $T = \boxed{35.5 \text{ kN}}$

(b) $\sum F_x = 0 \Rightarrow R_x - T \cos 71.1^\circ = 0$

or $R_x = (35.5 \text{ kN}) \cos 71.1^\circ = \boxed{11.5 \text{ kN (right)}}$

(c) $\sum F_y = 0 \Rightarrow R_y - 19.6 \text{ kN} + T \sin 71.1^\circ - 9.80 \text{ kN} = 0$

Thus,

$$\begin{aligned} R_y &= 29.4 \text{ kN} - (35.5 \text{ kN}) \sin 71.1^\circ = -4.19 \text{ kN} \\ &= \boxed{4.19 \text{ kN down}} \end{aligned}$$

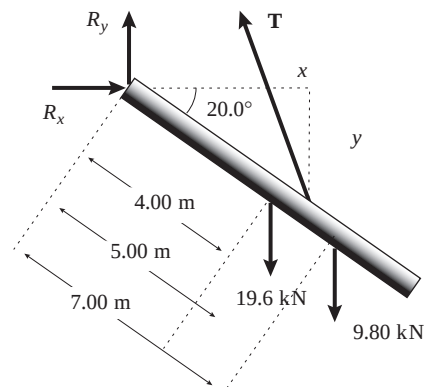


FIG. P12.20

- *P12.21** (a) We model the horse as a particle. The drawbridge will fall out from under the horse.

$$\begin{aligned}\alpha &= mg \frac{\frac{1}{2} \ell \cos \theta_0}{\frac{1}{3} m \ell^2} = \frac{3g}{2\ell} \cos \theta_0 \\ &= \frac{3(9.80 \text{ m/s}^2) \cos 20.0^\circ}{2(8.00 \text{ m})} = \boxed{1.73 \text{ rad/s}^2}\end{aligned}$$

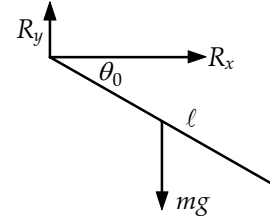


FIG. P12.21(a)

$$\begin{aligned}\text{(b)} \quad \frac{1}{2} I \omega^2 &= mgh \\ \therefore \frac{1}{2} \cdot \frac{1}{3} m \ell^2 \omega^2 &= mg \cdot \frac{1}{2} \ell (1 - \sin \theta_0) \\ \therefore \omega &= \sqrt{\frac{3g}{\ell} (1 - \sin \theta_0)} = \sqrt{\frac{3(9.80 \text{ m/s}^2)}{8.00 \text{ m}} (1 - \sin 20^\circ)} = \boxed{1.56 \text{ rad/s}}\end{aligned}$$

- (c) The linear acceleration of the bridge is:

$$a = \frac{1}{2} \ell \alpha = \frac{1}{2} (8.0 \text{ m}) (1.73 \text{ rad/s}^2) = 6.907 \text{ m/s}^2$$

The force at the hinge + the force of gravity produce the acceleration $a = 6.907 \text{ m/s}^2$ at right angles to the bridge.

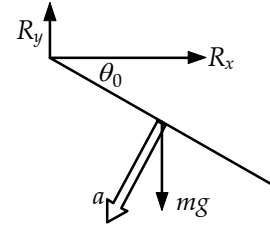


FIG. P12.21(c)

$$\begin{aligned}R_x &= ma_x = (2000 \text{ kg}) (6.907 \text{ m/s}^2) \cos 250^\circ = -4.72 \text{ kN} \\ R_y - mg &= ma_y \\ \therefore R_y &= m(g + a_y) = (2000 \text{ kg}) [9.80 \text{ m/s}^2 + (6.907 \text{ m/s}^2) \sin 250^\circ] = 6.62 \text{ kN}\end{aligned}$$

$$\text{Thus: } \boxed{\mathbf{R} = (-4.72\hat{\mathbf{i}} + 6.62\hat{\mathbf{j}}) \text{ kN}}.$$

$$\begin{aligned}\text{(d)} \quad R_x &= 0 \\ a &= \omega^2 \left(\frac{1}{2} \ell \right) = (1.56 \text{ rad/s})^2 (4.0 \text{ m}) = 9.67 \text{ m/s}^2 \\ R_y - mg &= ma \\ \therefore R_y &= (2000 \text{ kg}) (9.8 \text{ m/s}^2 + 9.67 \text{ m/s}^2) = 38.9 \text{ kN}\end{aligned}$$

$$\text{Thus: } \boxed{R_y = 38.9\hat{\mathbf{j}} \text{ kN}}$$

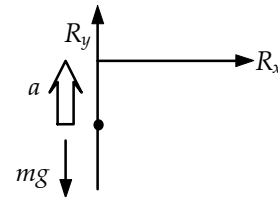


FIG. P12.21(d)

- P12.22** Call the required force F , with components $F_x = F \cos 15.0^\circ$ and $F_y = -F \sin 15.0^\circ$, transmitted to the center of the wheel by the handles.

Just as the wheel leaves the ground, the ground exerts no force on it.

$$\sum F_x = 0 : F \cos 15.0^\circ - n_x \quad (1)$$

$$\sum F_y = 0 : -F \sin 15.0^\circ - 400 \text{ N} + n_y = 0 \quad (2)$$

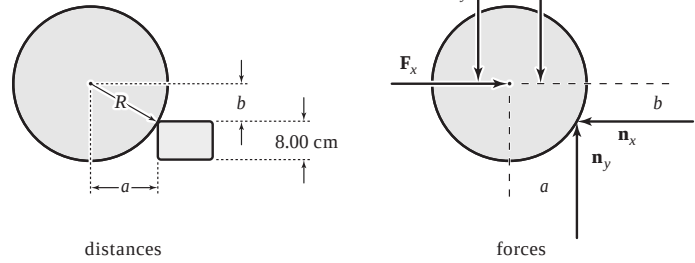


FIG. P12.22

Take torques about its contact point with the brick. The needed distances are seen to be:

$$b = R - 8.00 \text{ cm} = (20.0 - 8.00) \text{ cm} = 12.0 \text{ cm}$$

$$a = \sqrt{R^2 - b^2} = 16.0 \text{ cm}$$

$$(a) \quad \sum \tau = 0 : -F_x b + F_y a + (400 \text{ N})a = 0, \text{ or}$$

$$F[-(12.0 \text{ cm})\cos 15.0^\circ + (16.0 \text{ cm})\sin 15.0^\circ] + (400 \text{ N})(16.0 \text{ cm}) = 0$$

$$\text{so} \quad F = \frac{6\,400 \text{ N} \cdot \text{cm}}{7.45 \text{ cm}} = \boxed{859 \text{ N}}$$

(b) Then, using Equations (1) and (2),

$$n_x = (859 \text{ N})\cos 15.0^\circ = 830 \text{ N} \text{ and}$$

$$n_y = 400 \text{ N} + (859 \text{ N})\sin 15.0^\circ = 622 \text{ N}$$

$$n = \sqrt{n_x^2 + n_y^2} = \boxed{1.04 \text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{n_y}{n_x}\right) = \tan^{-1}(0.749) = \boxed{36.9^\circ \text{ to the left and upward}}$$

- *P12.23** When $x = x_{\min}$, the rod is on the verge of slipping, so

$$f = (f_s)_{\max} = \mu_s n = 0.50n.$$

$$\text{From } \sum F_x = 0, \quad n - T \cos 37^\circ = 0, \text{ or } n = 0.799T.$$

$$\text{Thus, } f = 0.50(0.799T) = 0.399T$$

$$\text{From } \sum F_y = 0, \quad f + T \sin 37^\circ - 2F_g = 0, \text{ or } 0.399T - 0.602T - 2F_g = 0, \text{ giving } T = 2.00F_g.$$

Using $\sum \tau = 0$ for an axis perpendicular to the page and through the left end of the beam gives

$$-F_g \cdot x_{\min} - F_g(2.0 \text{ m}) + [(2F_g)\sin 37^\circ](4.0 \text{ m}) = 0, \text{ which reduces to } x_{\min} = \boxed{2.82 \text{ m}}.$$

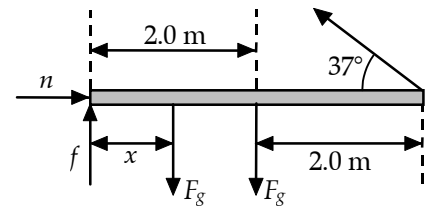


FIG. P12.23

P12.24 $x = \boxed{\frac{3L}{4}}$

If the CM of the two bricks does not lie over the edge, then the bricks balance.

If the lower brick is placed $\frac{L}{4}$ over the edge, then the second brick may be placed so that its end protrudes $\frac{3L}{4}$ over the edge.

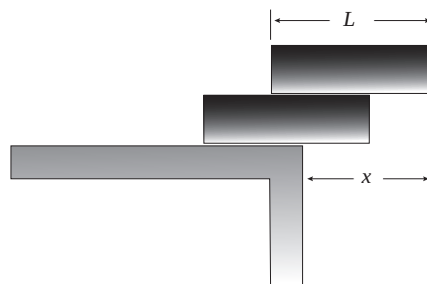


FIG. P12.24

P12.25 To find U , measure distances and forces from point A. Then, balancing torques,

$$(0.750)U = 29.4(2.25) \quad \boxed{U = 88.2 \text{ N}}$$

To find D , measure distances and forces from point B. Then, balancing torques,

$$(0.750)D = (1.50)(29.4) \quad \boxed{D = 58.8 \text{ N}}$$

Also, notice that $U = D + F_g$, so $\sum F_y = 0$.

***P12.26** Consider forces and torques on the beam.

$$\sum F_x = 0: \quad R \cos \theta - T \cos 53^\circ = 0$$

$$\sum F_y = 0: \quad R \sin \theta + T \sin 53^\circ - 800 \text{ N} = 0$$

$$\sum \tau = 0: \quad (T \sin 53^\circ)8 \text{ m} - (600 \text{ N})x - (200 \text{ N})4 \text{ m} = 0$$

(a) Then $T = \frac{600 \text{ N}x + 800 \text{ N} \cdot \text{m}}{8 \text{ m} \sin 53^\circ} = (93.9 \text{ N/m})x + 125 \text{ N}$. As x increases from 2 m, this expression grows larger.

(b) From substituting back,

$$R \cos \theta = [93.9x + 125] \cos 53^\circ$$

$$R \sin \theta = 800 \text{ N} - [93.9x + 125] \sin 53^\circ$$

$$\text{Dividing, } \tan \theta = \frac{R \sin \theta}{R \cos \theta} = -\tan 53^\circ + \frac{800 \text{ N}}{(93.9x + 125) \cos 53^\circ}$$

$$\tan \theta = \tan 53^\circ \left(\frac{32}{3x + 4} - 1 \right)$$

As x increases the fraction decreases and $\boxed{\theta \text{ decreases}}$.

continued on next page

- (c) To find R we can work out $R^2 \cos^2 \theta + R^2 \sin^2 \theta = R^2$. From the expressions above for $R \cos \theta$ and $R \sin \theta$,

$$R^2 = T^2 \cos^2 53^\circ + T^2 \sin^2 53^\circ - 1600 NT \sin 53^\circ + (800 \text{ N})^2$$

$$R^2 = T^2 - 1600T \sin 53^\circ + 640\,000$$

$$R^2 = (93.9x + 125)^2 - 1278(93.9x + 125) + 640\,000$$

$$R = (8819x^2 - 96482x + 495678)^{1/2}$$

At $x = 0$ this gives $R = 704 \text{ N}$. At $x = 2 \text{ m}$, $R = 581 \text{ N}$. At $x = 8 \text{ m}$, $R = 537 \text{ N}$. Over the range of possible values for x , the negative term $-96482x$ dominates the positive term $8819x^2$, and R decreases as x increases.

Section 12.4 Elastic Properties of Solids

P12.27 $\frac{F}{A} = Y \frac{\Delta L}{L_i}$

$$\Delta L = \frac{FL_i}{AY} = \frac{(200)(9.80)(4.00)}{(0.200 \times 10^{-4})(8.00 \times 10^{10})} = 4.90 \text{ mm}$$

P12.28 (a) $\text{stress} = \frac{F}{A} = \frac{F}{\pi r^2}$

$$F = (\text{stress})\pi \left(\frac{d}{2}\right)^2$$

$$F = (1.50 \times 10^8 \text{ N/m}^2)\pi \left(\frac{2.50 \times 10^{-2} \text{ m}}{2}\right)^2$$

$$F = 73.6 \text{ kN}$$

(b) $\text{stress} = Y(\text{strain}) = \frac{Y\Delta L}{L_i}$

$$\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{(1.50 \times 10^8 \text{ N/m}^2)(0.250 \text{ m})}{1.50 \times 10^{10} \text{ N/m}^2} = 2.50 \text{ mm}$$

***P12.29** The definition of $Y = \frac{\text{stress}}{\text{strain}}$ means that Y is the slope of the graph:

$$Y = \frac{300 \times 10^6 \text{ N/m}^2}{0.003} = 1.0 \times 10^{11} \text{ N/m}^2.$$

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- P12.30** Count the wires. If they are wrapped together so that all support nearly equal stress, the number should be

$$\frac{20.0 \text{ kN}}{0.200 \text{ kN}} = 100.$$

Since cross-sectional area is proportional to diameter squared, the diameter of the cable will be

$$(1 \text{ mm})\sqrt{100} \boxed{\sim 1 \text{ cm}}.$$

- P12.31** From the defining equation for the shear modulus, we find Δx as

$$\Delta x = \frac{hf}{SA} = \frac{(5.00 \times 10^{-3} \text{ m})(20.0 \text{ N})}{(3.0 \times 10^6 \text{ N/m}^2)(14.0 \times 10^{-4} \text{ m}^2)} = 2.38 \times 10^{-5} \text{ m}$$

or $\Delta x = \boxed{2.38 \times 10^{-2} \text{ mm}}.$

- P12.32** The force acting on the hammer changes its momentum according to

$$mv_i + \bar{F}(\Delta t) = mv_f \text{ so } |\bar{F}| = \frac{m|v_f - v_i|}{\Delta t}.$$

Hence, $|\bar{F}| = \frac{30.0 \text{ kg}|-10.0 \text{ m/s} - 20.0 \text{ m/s}|}{0.110 \text{ s}} = 8.18 \times 10^3 \text{ N}.$

By Newton's third law, this is also the magnitude of the average force exerted on the spike by the hammer during the blow. Thus, the stress in the spike is:

$$\text{stress} = \frac{F}{A} = \frac{8.18 \times 10^3 \text{ N}}{\pi \frac{(0.0230 \text{ m})^2}{4}} = 1.97 \times 10^7 \text{ N/m}^2$$

and the strain is: $\text{strain} = \frac{\text{stress}}{Y} = \frac{1.97 \times 10^7 \text{ N/m}^2}{20.0 \times 10^{10} \text{ N/m}^2} = \boxed{9.85 \times 10^{-5}}.$

P12.33 (a) $F = (A)(\text{stress})$
 $= \pi(5.00 \times 10^{-3} \text{ m})^2(4.00 \times 10^8 \text{ N/m}^2)$
 $= \boxed{3.14 \times 10^4 \text{ N}}$

- (b) The area over which the shear occurs is equal to the circumference of the hole times its thickness. Thus,

$$A = (2\pi r)t = 2\pi(5.00 \times 10^{-3} \text{ m})(5.00 \times 10^{-3} \text{ m})$$

$$= 1.57 \times 10^{-4} \text{ m}^2$$

So, $F = (A)\text{Stress} = (1.57 \times 10^{-4} \text{ m}^2)(4.00 \times 10^8 \text{ N/m}^2) = \boxed{6.28 \times 10^4 \text{ N}}.$

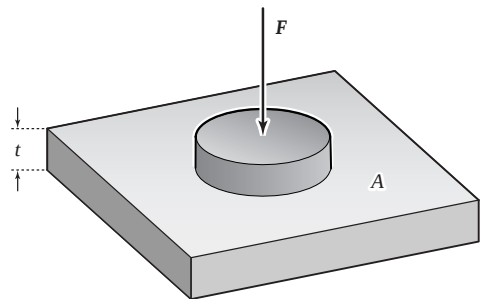


FIG. P12.33

- P12.34** Let the 3.00 kg mass be mass #1, with the 5.00 kg mass, mass # 2. Applying Newton's second law to each mass gives:

$$m_1 a = T - m_1 g \quad (1) \quad \text{and} \quad m_2 a = m_2 g - T \quad (2)$$

where T is the tension in the wire.

Solving equation (1) for the acceleration gives: $a = \frac{T}{m_1} - g$,

and substituting this into equation (2) yields: $\frac{m_2}{m_1} T - m_2 g = m_2 g - T$.

Solving for the tension T gives

$$T = \frac{2m_1 m_2 g}{m_2 + m_1} = \frac{2(3.00 \text{ kg})(5.00 \text{ kg})(9.80 \text{ m/s}^2)}{8.00 \text{ kg}} = 36.8 \text{ N}.$$

From the definition of Young's modulus, $Y = \frac{FL_i}{A(\Delta L)}$, the elongation of the wire is:

$$\Delta L = \frac{TL_i}{YA} = \frac{(36.8 \text{ N})(2.00 \text{ m})}{(2.00 \times 10^{11} \text{ N/m}^2)\pi(2.00 \times 10^{-3} \text{ m})^2} = \boxed{0.0293 \text{ mm}}.$$

- P12.35** Consider recompressing the ice, which has a volume $1.09V_0$.

$$\Delta P = -B \left(\frac{\Delta V}{V_i} \right) = \frac{-(2.00 \times 10^9 \text{ N/m}^2)(-0.090)}{1.09} = \boxed{1.65 \times 10^8 \text{ N/m}^2}$$

***P12.36** $B = -\frac{\Delta P}{\frac{\Delta V}{V_i}} = -\frac{\Delta P V_i}{\Delta V}$

$$(a) \quad \Delta V = -\frac{\Delta P V_i}{B} = -\frac{(1.13 \times 10^8 \text{ N/m}^2)1 \text{ m}^3}{0.21 \times 10^{10} \text{ N/m}^2} = \boxed{-0.0538 \text{ m}^3}$$

(b) The quantity of water with mass $1.03 \times 10^3 \text{ kg}$ occupies volume at the bottom $1 \text{ m}^3 - 0.0538 \text{ m}^3 = 0.946 \text{ m}^3$. So its density is $\frac{1.03 \times 10^3 \text{ kg}}{0.946 \text{ m}^3} = \boxed{1.09 \times 10^3 \text{ kg/m}^3}$.

- (c) With only a 5% volume change in this extreme case, liquid water is indeed nearly incompressible.

- *P12.37** Part of the load force extends the cable and part compresses the column by the same distance $\Delta \ell$:

$$F = \frac{Y_A A_A \Delta \ell}{\ell_A} + \frac{Y_s A_s \Delta \ell}{\ell_s}$$

$$\Delta \ell = \frac{F}{\frac{Y_A A_A}{\ell_A} + \frac{Y_s A_s}{\ell_s}} = \frac{8500 \text{ N}}{\frac{7 \times 10^{10} \pi (0.1624^2 - 0.1614^2)}{4(3.25)} + \frac{20 \times 10^{10} \pi (0.0127)^2}{4(5.75)}}$$

$$= \boxed{8.60 \times 10^{-4} \text{ m}}$$

Additional Problems

- *P12.38 (a) The beam is perpendicular to the wall, since $3^2 + 4^2 = 5^2$. Then $\sin \theta = \frac{4 \text{ m}}{5 \text{ m}}$; $\theta = 53.1^\circ$.

(b) $\sum \tau_{\text{hinge}} = 0: +T \sin \theta (3 \text{ m}) - 250 \text{ N}(10 \text{ m}) = 0$

$$T = \frac{2500 \text{ Nm}}{3 \text{ m} \sin 53.1^\circ} = 1.04 \times 10^3 \text{ N}$$

(c) $x = \frac{T}{k} = \frac{1.04 \times 10^3 \text{ N}}{8.25 \times 10^3 \text{ N/m}} = 0.126 \text{ m}$

The cable is 5.126 m long. From the law of cosines,

$$4^2 = 5.126^2 + 3^2 - 2(3)(5.126) \cos \theta$$

$$\theta = \cos^{-1} \frac{3^2 + 5.126^2 - 4^2}{2(3)(5.126)} = 51.2^\circ$$

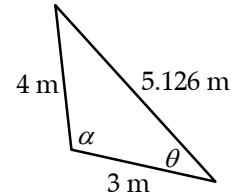


FIG. P12.38

- (d) From the law of sines, the angle the hinge makes with the wall satisfies $\frac{\sin \alpha}{5.126 \text{ m}} = \frac{\sin 51.2^\circ}{4 \text{ m}}$

$$\sin \alpha = 0.99858$$

$$\sum \tau_{\text{hinge}} = 0$$

$$+T(3 \text{ m}) \sin 51.2^\circ - 250 \text{ N}(10 \text{ m})(0.99858) = 0$$

$$T = 1.07 \times 10^3 \text{ N}$$

(e) $x = \frac{1.07 \times 10^3 \text{ N}}{8.25 \times 10^3 \text{ N/m}} = 0.129 \text{ m}$

$$\theta = \cos^{-1} \frac{3^2 + 5.129^2 - 4^2}{2(3)(5.129)} = 51.1^\circ$$

- (f) Now the answers are self-consistent:

$$\sin \alpha = 5.129 \text{ m} \frac{\sin 51.1^\circ}{4 \text{ m}} = 0.99851$$

$$T(3 \text{ m}) \sin 51.1^\circ - 250 \text{ N}(10 \text{ m})(0.99851) = 0$$

$$T = 1.07 \times 10^3 \text{ N}$$

$$x = 0.1295 \text{ m}$$

$$\theta = 51.1^\circ$$

- P12.39 Let n_A and n_B be the normal forces at the points of support.

Choosing the origin at point A with $\sum F_y = 0$ and $\sum \tau = 0$, we find:

$$n_A + n_B - (8.00 \times 10^4)g - (3.00 \times 10^4)g = 0 \text{ and}$$

$$-(3.00 \times 10^4)(g)15.0 - (8.00 \times 10^4)(g)25.0 + n_B(50.0) = 0$$

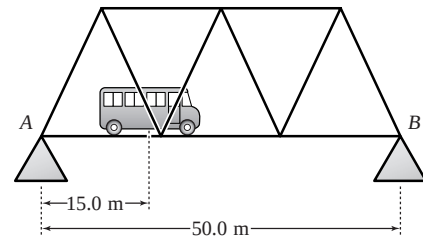


FIG. P12.39

The equations combine to give $n_A = 5.98 \times 10^5 \text{ N}$ and $n_B = 4.80 \times 10^5 \text{ N}$.

P12.40 When the concrete has cured and the pre-stressing tension has been released, the rod presses in on the concrete and with equal force, T_2 , the concrete produces tension in the rod.

(a) In the concrete: stress $= 8.00 \times 10^6 \text{ N/m}^2 = Y \cdot (\text{strain}) = Y \left(\frac{\Delta L}{L_i} \right)$

$$\text{Thus, } \Delta L = \frac{(\text{stress})L_i}{Y} = \frac{(8.00 \times 10^6 \text{ N/m}^2)(1.50 \text{ m})}{30.0 \times 10^9 \text{ N/m}^2}$$

or $\Delta L = 4.00 \times 10^{-4} \text{ m} = \boxed{0.400 \text{ mm}}$.

(b) In the concrete: stress $= \frac{T_2}{A_c} = 8.00 \times 10^6 \text{ N/m}^2$, so

$$T_2 = (8.00 \times 10^6 \text{ N/m}^2)(50.0 \times 10^{-4} \text{ m}^2) = \boxed{40.0 \text{ kN}}$$

(c) For the rod: $\frac{T_2}{A_R} = \left(\frac{\Delta L}{L_i} \right) Y_{\text{steel}}$ so $\Delta L = \frac{T_2 L_i}{A_R Y_{\text{steel}}}$

$$\Delta L = \frac{(40.0 \times 10^3 \text{ N})(1.50 \text{ m})}{(1.50 \times 10^{-4} \text{ m}^2)(20.0 \times 10^{10} \text{ N/m}^2)} = 2.00 \times 10^{-3} \text{ m} = \boxed{2.00 \text{ mm}}$$

(d) The rod in the finished concrete is 2.00 mm longer than its unstretched length. To remove stress from the concrete, one must stretch the rod 0.400 mm farther, by a total of $\boxed{2.40 \text{ mm}}$.

(e) For the stretched rod around which the concrete is poured:

$$\frac{T_1}{A_R} = \left(\frac{\Delta L_{\text{total}}}{L_i} \right) Y_{\text{steel}} \quad \text{or} \quad T_1 = \left(\frac{\Delta L_{\text{total}}}{L_i} \right) A_R Y_{\text{steel}}$$

$$T_1 = \left(\frac{2.40 \times 10^{-3} \text{ m}}{1.50 \text{ m}} \right) (1.50 \times 10^{-4} \text{ m}^2) (20.0 \times 10^{10} \text{ N/m}^2) = \boxed{48.0 \text{ kN}}$$

***P12.41** With ℓ as large as possible, n_1 and n_2 will both be large. The equality sign in $f_2 \leq \mu_s n_2$ will be true, but the less-than sign in $f_1 < \mu_s n_1$. Take torques about the lower end of the pole.

$$n_2 \ell \cos \theta + F_g \left(\frac{1}{2} \ell \right) \cos \theta - f_2 \ell \sin \theta = 0$$

Setting $f_2 = 0.576 n_2$, the torque equation becomes

$$n_2 (1 - 0.576 \tan \theta) + \frac{1}{2} F_g = 0$$

Since $n_2 > 0$, it is necessary that

$$1 - 0.576 \tan \theta < 0$$

$$\therefore \tan \theta > \frac{1}{0.576} = 1.736$$

$$\therefore \theta > 60.1^\circ$$

$$\therefore \ell = \frac{d}{\sin \theta} < \frac{7.80 \text{ ft}}{\sin 60.1^\circ} = \boxed{9.00 \text{ ft}}$$

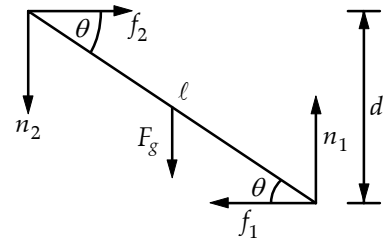


FIG. P12.41

P12.42 Call the normal forces A and B . They make angles α and β with the vertical.

$$\begin{aligned}\sum F_x = 0: & A \sin \alpha - B \sin \beta = 0 \\ \sum F_y = 0: & A \cos \alpha - Mg + B \cos \beta = 0\end{aligned}$$

Substitute $B = \frac{A \sin \alpha}{\sin \beta}$

$$\begin{aligned}A \cos \alpha + A \cos \beta \frac{\sin \alpha}{\sin \beta} &= Mg \\ A(\cos \alpha \sin \beta + \sin \alpha \cos \beta) &= Mg \sin \beta\end{aligned}$$

$$A = Mg \frac{\sin \beta}{\sin(\alpha + \beta)}$$

$$B = Mg \frac{\sin \alpha}{\sin(\alpha + \beta)}$$

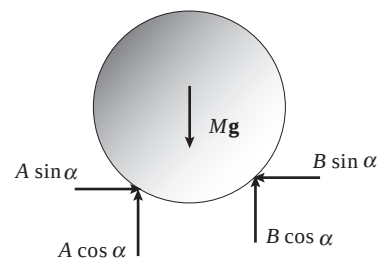
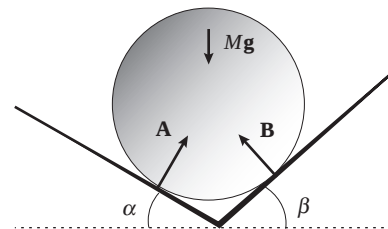


FIG. P12.42

P12.43 (a) See the diagram.

(b) If $x = 1.00$ m, then

$$\begin{aligned}\sum \tau_O &= (-700 \text{ N})(1.00 \text{ m}) - (200 \text{ N})(3.00 \text{ m}) \\ &\quad - (80.0 \text{ N})(6.00 \text{ m}) \\ &\quad + (T \sin 60.0^\circ)(6.00 \text{ m}) = 0\end{aligned}$$

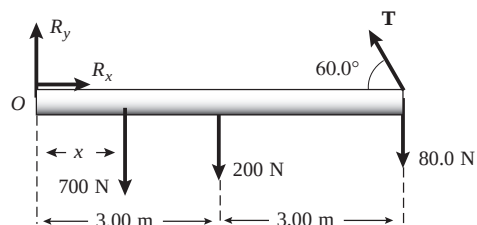


FIG. P12.43

Solving for the tension gives: $T = 343 \text{ N}$.

From $\sum F_x = 0$, $R_x = T \cos 60.0^\circ = 171 \text{ N}$.

From $\sum F_y = 0$, $R_y = 980 \text{ N} - T \sin 60.0^\circ = 683 \text{ N}$.

(c) If $T = 900 \text{ N}$:

$$\sum \tau_O = (-700 \text{ N})x - (200 \text{ N})(3.00 \text{ m}) - (80.0 \text{ N})(6.00 \text{ m}) + [(900 \text{ N}) \sin 60.0^\circ](6.00 \text{ m}) = 0.$$

Solving for x gives: $x = 5.13 \text{ m}$.

P12.44 (a) Sum the torques about top hinge:

$$\sum \tau = 0:$$

$$\begin{aligned} C(0) + D(0) + 200 \text{ N} \cos 30.0^\circ (0) \\ + 200 \text{ N} \sin 30.0^\circ (3.00 \text{ m}) \\ - 392 \text{ N}(1.50 \text{ m}) + A(1.80 \text{ m}) \\ + B(0) = 0 \end{aligned}$$

$$\text{Giving } A = \boxed{160 \text{ N (right)}}.$$

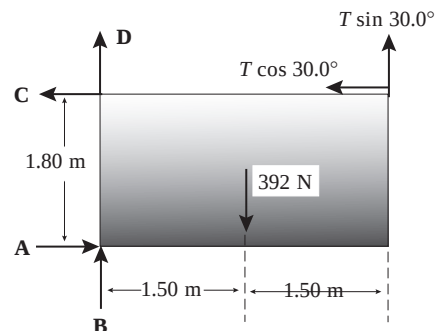


FIG. P12.44

(b) $\sum F_x = 0:$

$$\begin{aligned} -C - 200 \text{ N} \cos 30.0^\circ + A &= 0 \\ C &= 160 \text{ N} - 173 \text{ N} = -13.2 \text{ N} \end{aligned}$$

In our diagram, this means $\boxed{13.2 \text{ N to the right}}$.

(c) $\sum F_y = 0: +B + D - 392 \text{ N} + 200 \text{ N} \sin 30.0^\circ = 0$

$$B + D = 392 \text{ N} - 100 \text{ N} = \boxed{292 \text{ N (up)}}$$

(d) Given $C = 0$: Take torques about bottom hinge to obtain

$$A(0) + B(0) + 0(1.80 \text{ m}) + D(0) - 392 \text{ N}(1.50 \text{ m}) + T \sin 30.0^\circ (3.00 \text{ m}) + T \cos 30.0^\circ (1.80 \text{ m}) = 0$$

$$\text{so } T = \frac{588 \text{ N} \cdot \text{m}}{(1.50 \text{ m} + 1.56 \text{ m})} = \boxed{192 \text{ N}}.$$

P12.45 Using $\sum F_x = \sum F_y = \sum \tau = 0$, choosing the origin at the left end of the beam, we have (neglecting the weight of the beam)

$$\begin{aligned} \sum F_x &= R_x - T \cos \theta = 0, \\ \sum F_y &= R_y + T \sin \theta - F_g = 0, \end{aligned}$$

$$\text{and } \sum \tau = -F_g(L + d) + T \sin \theta(2L + d) = 0.$$

Solving these equations, we find:

$$(a) \quad T = \boxed{\frac{F_g(L + d)}{\sin \theta(2L + d)}}$$

$$(b) \quad R_x = \boxed{\frac{F_g(L + d) \cot \theta}{2L + d}} \quad R_y = \boxed{\frac{F_g L}{2L + d}}$$

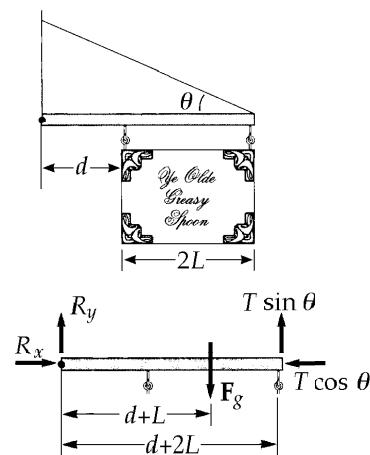


FIG. P12.45

P12.46 $\sum \tau_{\text{point } 0} = 0$ gives

$$(T \cos 25.0^\circ) \left(\frac{3\ell}{4} \sin 65.0^\circ \right) + (T \sin 25.0^\circ) \left(\frac{3\ell}{4} \cos 65.0^\circ \right) \\ = (2000 \text{ N})(\ell \cos 65.0^\circ) + (1200 \text{ N}) \left(\frac{\ell}{2} \cos 65.0^\circ \right)$$

From which, $T = 1465 \text{ N} = \boxed{1.46 \text{ kN}}$

From $\sum F_x = 0$,

$$H = T \cos 25.0^\circ = 1328 \text{ N (toward right)} = \boxed{1.33 \text{ kN}}$$

From $\sum F_y = 0$,

$$V = 3200 \text{ N} - T \sin 25.0^\circ = 2581 \text{ N (upward)} = \boxed{2.58 \text{ kN}}$$

P12.47 We interpret the problem to mean that the support at point B is frictionless. Then the support exerts a force in the x direction and

$$\boxed{F_{By} = 0}$$

$$\sum F_x = F_{Bx} - F_{Ax} = 0$$

$$F_{Ay} - (3000 + 10000)g = 0$$

$$\text{and } \sum \tau = -(3000g)(2.00) - (10000g)(6.00) + F_{Bx}(1.00) = 0.$$

These equations combine to give

$$F_{Ax} = F_{Bx} = \boxed{6.47 \times 10^5 \text{ N}}$$

$$F_{Ay} = \boxed{1.27 \times 10^5 \text{ N}}$$

P12.48 $n = (M + m)g$ $H = f$

$$H_{\max} = f_{\max} = \mu_s(m + M)g$$

$$\sum \tau_A = 0 = \frac{mgL}{2} \cos 60.0^\circ + Mgx \cos 60.0^\circ - HL \sin 60.0^\circ$$

$$\frac{x}{L} = \frac{H \tan 60.0^\circ}{Mg} - \frac{m}{2M} = \frac{\mu_s(m + M) \tan 60.0^\circ}{M} - \frac{m}{2M} \\ = \frac{3}{2} \mu_s \tan 60.0^\circ - \frac{1}{4} = \boxed{0.789}$$

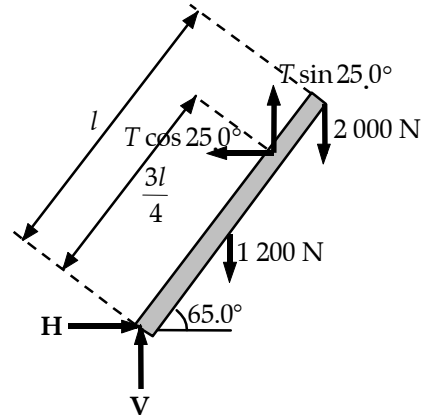


FIG. P12.46

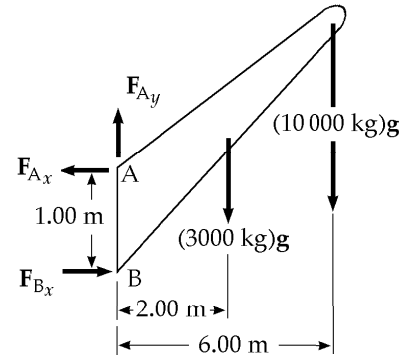


FIG. P12.47

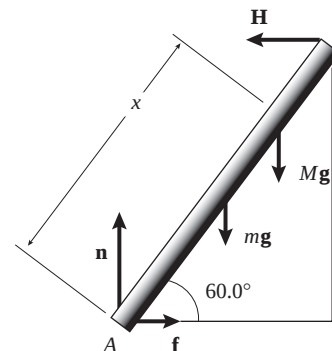


FIG. P12.48

P12.49 From the free-body diagram, the angle T makes with the rod is

$$\theta = 60.0^\circ + 20.0^\circ = 80.0^\circ$$

and the perpendicular component of T is $T \sin 80.0^\circ$.

Summing torques around the base of the rod,

$$\sum \tau = 0: \quad -(4.00 \text{ m})(10\,000 \text{ N}) \cos 60.0^\circ + T(4.00 \text{ m}) \sin 80.0^\circ = 0$$

$$T = \frac{(10\,000 \text{ N}) \cos 60.0^\circ}{\sin 80.0^\circ} = \boxed{5.08 \times 10^3 \text{ N}}$$

$$\sum F_x = 0: \quad F_H - T \cos 20.0^\circ = 0$$

$$F_H = T \cos 20.0^\circ = \boxed{4.77 \times 10^3 \text{ N}}$$

$$\sum F_y = 0: \quad F_V + T \sin 20.0^\circ - 10\,000 \text{ N} = 0$$

$$\text{and } F_V = (10\,000 \text{ N}) - T \sin 20.0^\circ = \boxed{8.26 \times 10^3 \text{ N}}$$

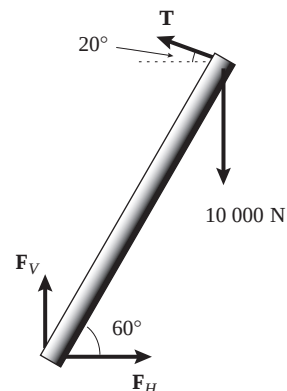


FIG. P12.49

P12.50 Choosing the origin at R ,

$$(1) \quad \sum F_x = +R \sin 15.0^\circ - T \sin \theta = 0$$

$$(2) \quad \sum F_y = 700 - R \cos 15.0^\circ + T \cos \theta = 0$$

$$(3) \quad \sum \tau = -700 \cos \theta (0.180) + T(0.070) = 0$$

Solve the equations for θ

$$\text{from (3), } T = 1\,800 \cos \theta \text{ from (1), } R = \frac{1\,800 \sin \theta \cos \theta}{\sin 15.0^\circ}$$

$$\text{Then (2) gives } 700 - \frac{1\,800 \sin \theta \cos \theta \cos 15.0^\circ}{\sin 15.0^\circ} + 1\,800 \cos^2 \theta = 0$$

$$\text{or } \cos^2 \theta + 0.3889 - 3.732 \sin \theta \cos \theta = 0$$

$$\text{Squaring, } \cos^4 \theta - 0.8809 \cos^2 \theta + 0.01013 = 0$$

$$\text{Let } u = \cos^2 \theta \text{ then using the quadratic equation,} \\ u = 0.01165 \text{ or } 0.8693$$

Only the second root is physically possible,

$$\therefore \theta = \cos^{-1} \sqrt{0.8693} = \boxed{21.2^\circ}$$

$$\therefore T = \boxed{1.68 \times 10^3 \text{ N}} \quad \text{and} \quad R = \boxed{2.34 \times 10^3 \text{ N}}$$

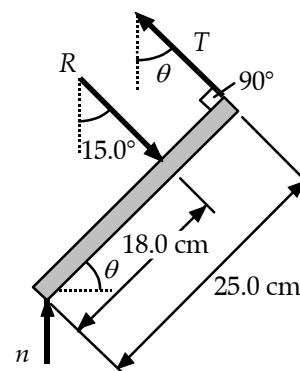


FIG. P12.50

P12.51 Choosing torques about R , with $\sum \tau = 0$

$$-\frac{L}{2}(350 \text{ N}) + (T \sin 12.0^\circ) \left(\frac{2L}{3} \right) - (200 \text{ N})L = 0.$$

$$\text{From which, } T = \boxed{2.71 \text{ kN}}.$$

Let R_x = compression force along spine, and from $\sum F_x = 0$

$$R_x = T_x = T \cos 12.0^\circ = \boxed{2.65 \text{ kN}}.$$

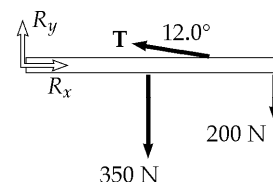


FIG. P12.51

- P12.52** (a) Just three forces act on the rod: forces perpendicular to the sides of the trough at A and B, and its weight. The lines of action of A and B will intersect at a point above the rod. They will have no torque about this point. The rod's weight will cause a torque about the point of intersection as in Figure 12.52(a), and the rod will not be in equilibrium unless the center of the rod lies vertically below the intersection point, as in Figure 12.52(b). All three forces must be concurrent. Then the line of action of the weight is a diagonal of the rectangle formed by the trough and the normal forces, and the rod's center of gravity is vertically above the bottom of the trough.

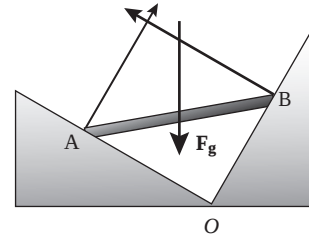


FIG. P12.52(a)

- (b) In Figure (b), $\overline{AO} \cos 30.0^\circ = \overline{BO} \cos 60.0^\circ$ and

$$L^2 = \overline{AO}^2 + \overline{BO}^2 = \overline{AO}^2 + \overline{AO}^2 \left(\frac{\cos^2 30.0^\circ}{\cos^2 60.0^\circ} \right)$$

$$\overline{AO} = \frac{L}{\sqrt{1 + \frac{\cos^2 30.0^\circ}{\cos^2 60.0^\circ}}} = \frac{L}{2}$$

$$\text{So } \cos \theta = \frac{\overline{AO}}{L} = \frac{1}{2} \text{ and } \theta = \boxed{60.0^\circ}.$$

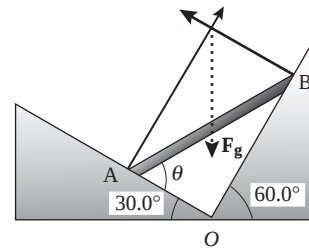


FIG. P12.52(b)

- P12.53** (a) Locate the origin at the bottom left corner of the cabinet and let x = distance between the *resultant normal force* and the *front of the cabinet*. Then we have

$$\sum F_x = 200 \cos 37.0^\circ - \mu n = 0 \quad (1)$$

$$\sum F_y = 200 \sin 37.0^\circ + n - 400 = 0 \quad (2)$$

$$\sum \tau = n(0.600 - x) - 400(0.300) + 200 \sin 37.0^\circ (0.600) - 200 \cos 37.0^\circ (0.400) = 0 \quad (3)$$

$$\text{From (2), } n = 400 - 200 \sin 37.0^\circ = 280 \text{ N}$$

$$\text{From (3), } x = \frac{72.2 - 120 + 280(0.600) - 64.0}{280}$$

$$x = \boxed{20.1 \text{ cm}} \text{ to the left of the front edge}$$

$$\text{From (1), } \mu_k = \frac{200 \cos 37.0^\circ}{280} = \boxed{0.571}$$

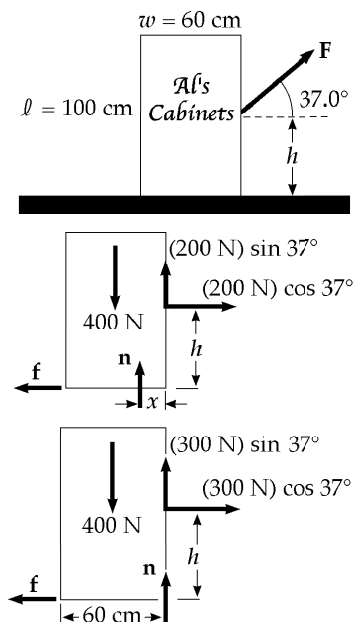


FIG. P12.53

- (b) In this case, locate the origin $x = 0$ at the bottom right corner of the cabinet. Since the cabinet is about to tip, we can use $\sum \tau = 0$ to find h :

$$\sum \tau = 400(0.300) - (300 \cos 37.0^\circ)h = 0 \quad h = \frac{120}{300 \cos 37.0^\circ} = \boxed{0.501 \text{ m}}$$

- P12.54** (a), (b) Use the first diagram and sum the torques about the lower front corner of the cabinet.

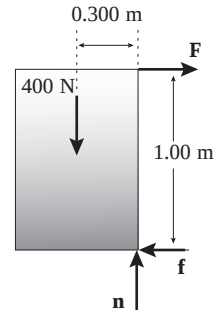
$$\sum \tau = 0 \Rightarrow -F(1.00 \text{ m}) + (400 \text{ N})(0.300 \text{ m}) = 0$$

$$\text{yielding } F = \frac{(400 \text{ N})(0.300 \text{ m})}{1.00 \text{ m}} = \boxed{120 \text{ N}}$$

$$\sum F_x = 0 \Rightarrow -f + 120 \text{ N} = 0, \quad \text{or} \quad f = 120 \text{ N}$$

$$\sum F_y = 0 \Rightarrow -400 \text{ N} + n = 0, \quad \text{so} \quad n = 400 \text{ N}$$

$$\text{Thus, } \mu_s = \frac{f}{n} = \frac{120 \text{ N}}{400 \text{ N}} = \boxed{0.300}.$$



- (c) Apply F' at the upper rear corner and directed so $\theta + \phi = 90.0^\circ$ to obtain the largest possible lever arm.

$$\theta = \tan^{-1}\left(\frac{1.00 \text{ m}}{0.600 \text{ m}}\right) = 59.0^\circ$$

$$\text{Thus, } \phi = 90.0^\circ - 59.0^\circ = 31.0^\circ.$$

Sum the torques about the lower front corner of the cabinet:

$$-F' \sqrt{(1.00 \text{ m})^2 + (0.600 \text{ m})^2} + (400 \text{ N})(0.300 \text{ m}) = 0$$

$$\text{so } F' = \frac{120 \text{ N} \cdot \text{m}}{1.17 \text{ m}} = 103 \text{ N}.$$

Therefore, the minimum force required to tip the cabinet is

$$\boxed{103 \text{ N applied at } 31.0^\circ \text{ above the horizontal at the upper left corner}}.$$

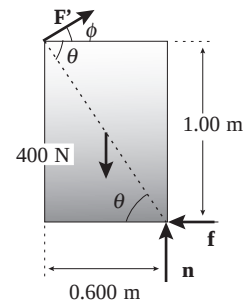


FIG. P12.54

- P12.55** (a) We can use $\sum F_x = \sum F_y = 0$ and $\sum \tau = 0$ with pivot point at the contact on the floor.

$$\text{Then } \sum F_x = T - \mu_s n = 0,$$

$$\sum F_y = n - Mg - mg = 0, \text{ and}$$

$$\sum \tau = Mg(L \cos \theta) + mg\left(\frac{L}{2} \cos \theta\right) - T(L \sin \theta) = 0$$

Solving the above equations gives

$$M = \left[\frac{m}{2} \left(\frac{2\mu_s \sin \theta - \cos \theta}{\cos \theta - \mu_s \sin \theta} \right) \right]$$

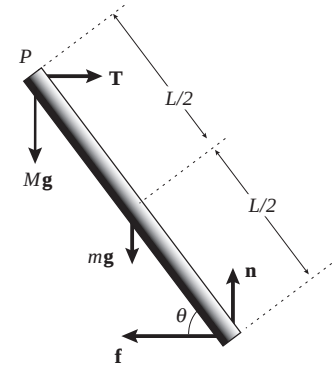


FIG. P12.55

This answer is the maximum value for M if $\mu_s < \cot \theta$. If $\mu_s \geq \cot \theta$, the mass M can increase without limit. It has no maximum value, and part (b) cannot be answered as stated either. In the case $\mu_s < \cot \theta$, we proceed.

- (b) At the floor, we have the normal force in the y -direction and frictional force in the x -direction. The reaction force then is

$$R = \sqrt{n^2 + (\mu_s n)^2} = \boxed{(M + m)g\sqrt{1 + \mu_s^2}}.$$

At point P, the force of the beam on the rope is

$$F = \sqrt{T^2 + (Mg)^2} = \boxed{g\sqrt{M^2 + \mu_s^2(M + m)^2}}.$$

P12.56 (a) The height of pin B is

$$(10.0 \text{ m}) \sin 30.0^\circ = 5.00 \text{ m}.$$

The length of bar BC is then

$$\overline{BC} = \frac{5.00 \text{ m}}{\sin 45.0^\circ} = 7.07 \text{ m}.$$

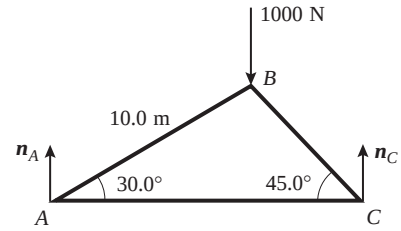


FIG. P12.56(a)

Consider the entire truss:

$$\sum F_y = n_A - 1000 \text{ N} + n_C = 0$$

$$\sum \tau_A = -(1000 \text{ N})(10.0 \cos 30.0^\circ) + n_C [10.0 \cos 30.0^\circ + 7.07 \cos 45.0^\circ] = 0$$

Which gives $n_C = 634 \text{ N}$.

Then, $n_A = 1000 \text{ N} - n_C = 366 \text{ N}$.

- (b) Suppose that a bar exerts on a pin a force not along the length of the bar. Then, the pin exerts on the bar a force with a component perpendicular to the bar. The only other force on the bar is the pin force on the other end. For $\sum \mathbf{F} = 0$, this force must also have a component perpendicular to the bar. Then, the total torque on the bar is not zero. The contradiction proves that the bar can only exert forces along its length.

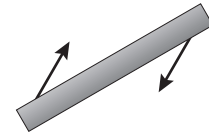


FIG. P12.56(b)

- (c) Joint A:

$$\sum F_y = 0: -C_{AB} \sin 30.0^\circ + 366 \text{ N} = 0,$$

so $C_{AB} = 732 \text{ N}$

$$\sum F_x = 0: -C_{AB} \cos 30.0^\circ + T_{AC} = 0$$

$$T_{AC} = (732 \text{ N}) \cos 30.0^\circ = 634 \text{ N}$$

Joint B:

$$\sum F_x = 0: (732 \text{ N}) \cos 30.0^\circ - C_{BC} \cos 45.0^\circ = 0$$

$$C_{BC} = \frac{(732 \text{ N}) \cos 30.0^\circ}{\cos 45.0^\circ} = 897 \text{ N}$$

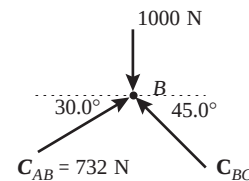
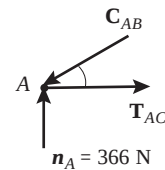


FIG. P12.56(c)

P12.57 From geometry, observe that

$$\cos \theta = \frac{1}{4} \quad \text{and} \quad \theta = 75.5^\circ$$

For the left half of the ladder, we have

$$\sum F_x = T - R_x = 0 \quad (1)$$

$$\sum F_y = R_y + n_A - 686 \text{ N} = 0 \quad (2)$$

$$\begin{aligned} \sum \tau_{\text{top}} &= 686 \text{ N}(1.00 \cos 75.5^\circ) + T(2.00 \sin 75.5^\circ) \\ &- n_A(4.00 \cos 75.5^\circ) = 0 \end{aligned} \quad (3)$$

For the right half of the ladder we have

$$\sum F_x = R_x - T = 0$$

$$\sum F_y = n_B - R_y = 0 \quad (4)$$

$$\sum \tau_{\text{top}} = n_B(4.00 \cos 75.5^\circ) - T(2.00 \sin 75.5^\circ) = 0 \quad (5)$$

Solving equations 1 through 5 simultaneously yields:

(a) $T = 133 \text{ N}$

(b) $n_A = 429 \text{ N}$ and $n_B = 257 \text{ N}$

(c) $R_x = 133 \text{ N}$ and $R_y = 257 \text{ N}$

The force exerted by the left half of the ladder on the right half is to the right and downward.

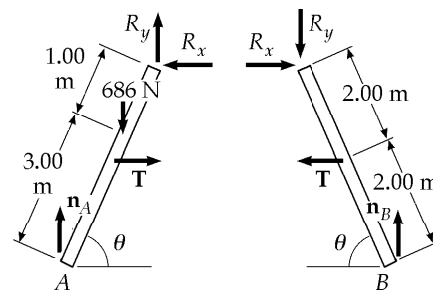


FIG. P12.57

P12.58 (a)
$$x_{\text{CG}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(1000 \text{ kg})10.0 \text{ m} + (125 \text{ kg})0 + (125 \text{ kg})0 + (125 \text{ kg})20.0 \text{ m}}{1375 \text{ kg}} = 9.09 \text{ m}$$

$$y_{\text{CG}} = \frac{(1000 \text{ kg})10.0 \text{ m} + (125 \text{ kg})20.0 \text{ m} + (125 \text{ kg})20.0 \text{ m} + (125 \text{ kg})0}{1375 \text{ kg}} = 10.9 \text{ m}$$

(b) By symmetry, $x_{\text{CG}} = 10.0 \text{ m}$

There is no change in $y_{\text{CG}} = 10.9 \text{ m}$

(c)
$$v_{\text{CG}} = \left(\frac{10.0 \text{ m} - 9.09 \text{ m}}{8.00 \text{ s}} \right) = 0.114 \text{ m/s}$$

P12.59 Considering the torques about the point at the bottom of the bracket yields:

$$(0.0500 \text{ m})(80.0 \text{ N}) - F(0.0600 \text{ m}) = 0 \quad \text{so} \quad F = 66.7 \text{ N}$$

P12.60 When it is on the verge of slipping, the cylinder is in equilibrium.

$$\sum F_x = 0: \quad f_1 = n_2 = \mu_s n_1 \quad \text{and} \quad f_2 = \mu_s n_2$$

$$\sum F_y = 0: \quad P + n_1 + f_2 = F_g$$

$$\sum \tau = 0: \quad P = f_1 + f_2$$

As P grows so do f_1 and f_2

$$\text{Therefore, since } \mu_s = \frac{1}{2}, \quad f_1 = \frac{n_1}{2} \quad \text{and} \quad f_2 = \frac{n_2}{2} = \frac{n_1}{4}$$

$$\text{then} \quad P + n_1 + \frac{n_1}{4} = F_g \quad (1) \quad \text{and} \quad P = \frac{n_1}{2} + \frac{n_1}{4} = \frac{3}{4} n_1 \quad (2)$$

$$\text{So} \quad P + \frac{5}{4} n_1 = F_g \quad \text{becomes} \quad P + \frac{5}{4} \left(\frac{4}{3} P \right) = F_g \quad \text{or} \quad \frac{8}{3} P = F_g$$

$$\text{Therefore, } P = \boxed{\frac{3}{8} F_g}$$

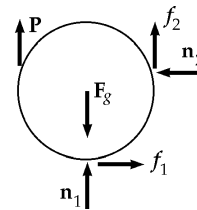


FIG. P12.60

P12.61 (a) $|F| = k(\Delta L)$, Young's modulus is $Y = \frac{F}{\frac{\Delta L}{A}} = \frac{FL_i}{A(\Delta L)}$

$$\text{Thus, } Y = \frac{kL_i}{A} \quad \text{and} \quad k = \boxed{\frac{YA}{L_i}}$$

(b) $W = -\int_0^{\Delta L} F dx = -\int_0^{\Delta L} (-kx) dx = \frac{YA}{L_i} \int_0^{\Delta L} x dx = \boxed{YA \frac{(\Delta L)^2}{2L_i}}$

P12.62 (a) Take both balls together. Their weight is 3.33 N and their CG is at their contact point.

$$\sum F_x = 0: +P_3 - P_1 = 0$$

$$\sum F_y = 0: +P_2 - 3.33 \text{ N} = 0 \quad P_2 = \boxed{3.33 \text{ N}}$$

$$\sum \tau_A = 0: -P_3 R + P_2 R - 3.33 \text{ N}(R + R \cos 45.0^\circ) + P_1 (R + 2R \cos 45.0^\circ) = 0$$

Substituting,

$$-P_1 R + (3.33 \text{ N})R - (3.33 \text{ N})R(1 + \cos 45.0^\circ) + P_1 R(1 + 2 \cos 45.0^\circ) = 0$$

$$(3.33 \text{ N}) \cos 45.0^\circ = 2P_1 \cos 45.0^\circ$$

$$P_1 = \boxed{1.67 \text{ N}} \quad \text{so} \quad P_3 = \boxed{1.67 \text{ N}}$$

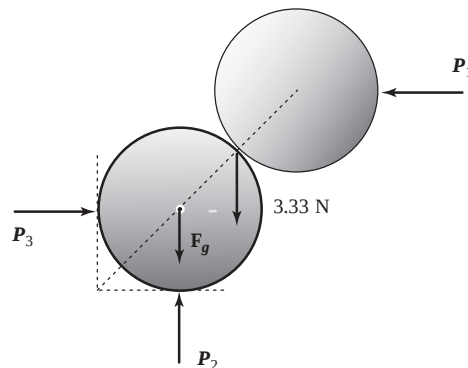


FIG. P12.62(a)

(b) Take the upper ball. The lines of action of its weight, of P_1 , and of the normal force n exerted by the lower ball all go through its center, so for rotational equilibrium there can be no frictional force.

$$\sum F_x = 0: n \cos 45.0^\circ - P_1 = 0$$

$$n = \frac{1.67 \text{ N}}{\cos 45.0^\circ} = \boxed{2.36 \text{ N}}$$

$$\sum F_y = 0: n \sin 45.0^\circ - 1.67 \text{ N} = 0 \quad \text{gives the same result}$$

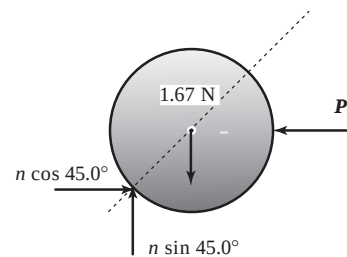


FIG. P12.62(b)

P12.63 $\sum F_y = 0:$ $+380 \text{ N} - F_g + 320 \text{ N} = 0$

$$F_g = 700 \text{ N}$$

Take torques about her feet:

$$\sum \tau = 0: \quad -380 \text{ N}(2.00 \text{ m}) + (700 \text{ N})x + (320 \text{ N})0 = 0$$

$$x = \boxed{1.09 \text{ m}}$$

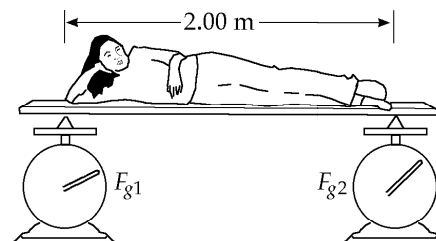


FIG. P12.63

P12.64 The tension in this cable is not uniform, so this becomes a fairly difficult problem.

$$\frac{dL}{L} = \frac{F}{YA}$$

At any point in the cable, F is the weight of cable below that point. Thus, $F = \mu gy$ where μ is the mass per unit length of the cable.

$$\text{Then, } \Delta y = \int_0^{L_i} \left(\frac{dL}{L} \right) dy = \frac{\mu g}{YA} \int_0^{L_i} y dy = \frac{1}{2} \frac{\mu g L_i^2}{YA}$$

$$\Delta y = \frac{1}{2} \frac{(2.40)(9.80)(500)^2}{(2.00 \times 10^{11})(3.00 \times 10^{-4})} = 0.0490 \text{ m} = \boxed{4.90 \text{ cm}}$$

P12.65 (a) $F = m \left(\frac{\Delta v}{\Delta t} \right) = (1.00 \text{ kg}) \frac{(10.0 - 1.00) \text{ m/s}}{0.002 \text{ s}} = \boxed{4500 \text{ N}}$

(b) $\text{stress} = \frac{F}{A} = \frac{4500 \text{ N}}{(0.010 \text{ m})(0.100 \text{ m})} = \boxed{4.50 \times 10^6 \text{ N/m}^2}$

(c) Yes. This is more than sufficient to break the board.

- P12.66** The CG lies above the center of the bottom. Consider a disk of water at height y above the bottom. Its radius is

$$25.0 \text{ cm} + (35.0 - 25.0 \text{ cm})\left(\frac{y}{30.0 \text{ cm}}\right) = 25.0 \text{ cm} + \frac{y}{3}$$

Its area is $\pi\left(25.0 \text{ cm} + \frac{y}{3}\right)^2$. Its volume is $\pi\left(25.0 \text{ cm} + \frac{y}{3}\right)^2 dy$ and its mass is $\pi\rho\left(25.0 \text{ cm} + \frac{y}{3}\right)^2 dy$. The whole mass of the water is

$$M = \int_{y=0}^{30.0 \text{ cm}} dm = \int_0^{30.0 \text{ cm}} \pi\rho\left(625 + \frac{50.0y}{3} + \frac{y^2}{9}\right) dy$$

$$M = \pi\rho\left[625y + \frac{50.0y^2}{6} + \frac{y^3}{27}\right]_0^{30.0}$$

$$M = \pi\rho\left[625(30.0) + \frac{50.0(30.0)^2}{6} + \frac{(30.0)^3}{27}\right]$$

$$M = \pi(10^{-3} \text{ kg/cm}^3)(27\,250 \text{ cm}^3) = 85.6 \text{ kg}$$

The height of the center of gravity is

$$\begin{aligned} y_{\text{CG}} &= \int_{y=0}^{30.0 \text{ cm}} \frac{y dm}{M} \\ &= \pi\rho \int_0^{30.0 \text{ cm}} \left(625y + \frac{50.0y^2}{3} + \frac{y^3}{9}\right) \frac{dy}{M} \\ &= \frac{\pi\rho}{M} \left[\frac{625y^2}{2} + \frac{50.0y^3}{9} + \frac{y^4}{36}\right]_0^{30.0 \text{ cm}} \\ &= \frac{\pi\rho}{M} \left[\frac{625(30.0)^2}{2} + \frac{50.0(30.0)^3}{9} + \frac{(30.0)^4}{36}\right] \\ &= \frac{\pi(10^{-3} \text{ kg/cm}^3)}{M} [453\,750 \text{ cm}^4] \\ y_{\text{CG}} &= \frac{1.43 \times 10^3 \text{ kg} \cdot \text{cm}}{85.6 \text{ kg}} = \boxed{16.7 \text{ cm}} \end{aligned}$$

- P12.67** Let θ represent the angle of the wire with the vertical. The radius of the circle of motion is $r = (0.850 \text{ m}) \sin \theta$.
For the mass:

$$\sum F_r = ma_r = m \frac{v^2}{r} = mr\omega^2$$

$$T \sin \theta = m[(0.850 \text{ m}) \sin \theta] \omega^2$$

Further, $\frac{T}{A} = Y \cdot (\text{strain})$ or $T = AY \cdot (\text{strain})$

Thus, $AY \cdot (\text{strain}) = m(0.850 \text{ m}) \omega^2$, giving

$$\omega = \sqrt{\frac{AY \cdot (\text{strain})}{m(0.850 \text{ m})}} = \sqrt{\frac{\pi(3.90 \times 10^{-4} \text{ m})^2 (7.00 \times 10^{10} \text{ N/m}^2)(1.00 \times 10^{-3})}{(1.20 \text{ kg})(0.850 \text{ m})}}$$

or $\omega = \boxed{5.73 \text{ rad/s}}$.

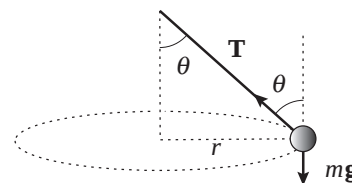


FIG. P12.67

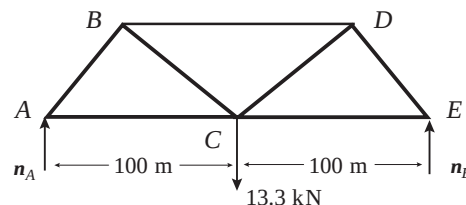
- P12.68** For the bridge as a whole:

$$\sum \tau_A = n_A(0) - (13.3 \text{ kN})(100 \text{ m}) + n_E(200 \text{ m}) = 0$$

so $n_E = \frac{(13.3 \text{ kN})(100 \text{ m})}{200 \text{ m}} = \boxed{6.66 \text{ kN}}$

$$\sum F_y = n_A - 13.3 \text{ kN} + n_E = 0 \text{ gives}$$

$$n_A = 13.3 \text{ kN} - n_E = \boxed{6.66 \text{ kN}}$$



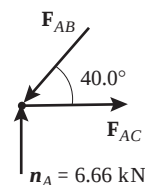
At Pin A:

$$\sum F_y = -F_{AB} \sin 40.0^\circ + 6.66 \text{ kN} = 0 \text{ or}$$

$$F_{AB} = \frac{6.66 \text{ kN}}{\sin 40.0^\circ} = \boxed{10.4 \text{ kN (compression)}}$$

$$\sum F_x = F_{AC} - (10.4 \text{ kN}) \cos 40.0^\circ = 0 \text{ so}$$

$$F_{AC} = (10.4 \text{ kN}) \cos 40.0^\circ = \boxed{7.94 \text{ kN (tension)}}$$



At Pin B:

$$\sum F_y = (10.4 \text{ kN}) \sin 40.0^\circ - F_{BC} \sin 40.0^\circ = 0$$

Thus, $F_{BC} = \boxed{10.4 \text{ kN (tension)}}$

$$\sum F_x = F_{AB} \cos 40.0^\circ + F_{BC} \cos 40.0^\circ - F_{BD} = 0$$

$$F_{BD} = 2(10.4 \text{ kN}) \cos 40.0^\circ = \boxed{15.9 \text{ kN (compression)}}$$

By symmetry: $F_{DE} = F_{AB} = 10.4 \text{ kN (compression)}$

$$F_{DC} = F_{BC} = 10.4 \text{ kN (tension)}$$

and $F_{EC} = F_{AC} = 7.94 \text{ kN (tension)}$

We can check by analyzing Pin C:

$$\sum F_x = +7.94 \text{ kN} - 7.94 \text{ kN} = 0 \text{ or } 0 = 0$$

$$\sum F_y = 2(10.4 \text{ kN}) \sin 40.0^\circ - 13.3 \text{ kN} = 0$$

which yields $0 = 0$.

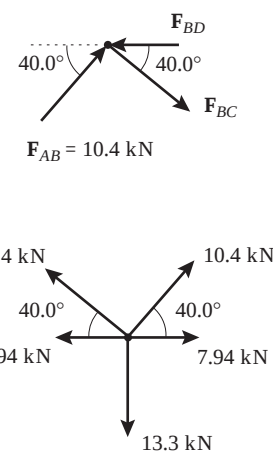


FIG. P12.68

P12.69 Member AC is not in pure compression or tension. It also has shear forces present. It exerts a downward force S_{AC} and a tension force F_{AC} on Pin A and on Pin C. Still, this member is in equilibrium.

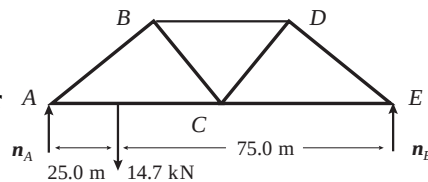
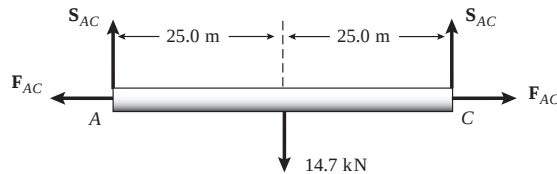
$$\sum F_x = F_{AC} - F'_{AC} = 0 \Rightarrow F_{AC} = F'_{AC}$$

$$\sum \tau_A = 0: \quad -(14.7 \text{ kN})(25.0 \text{ m}) + S'_{AC}(50.0 \text{ m}) = 0$$

or $S'_{AC} = 7.35 \text{ kN}$

$$\sum F_y = S_{AC} - 14.7 \text{ kN} + 7.35 \text{ kN} = 0 \Rightarrow S_{AC} = 7.35 \text{ kN}$$

Then $S_{AC} = S'_{AC}$ and we have proved that the loading by the car is equivalent to one-half the weight of the car pulling down on each of pins A and C, so far as the rest of the truss is concerned.



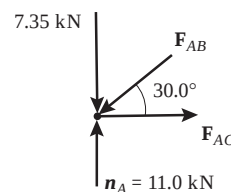
For the Bridge as a whole: $\sum \tau_A = 0$:

$$-(14.7 \text{ kN})(25.0 \text{ m}) + n_E(100 \text{ m}) = 0$$

$$n_E = 3.67 \text{ kN}$$

$$\sum F_y = n_A - 14.7 \text{ kN} + 3.67 \text{ kN} = 0$$

$$n_A = 11.0 \text{ kN}$$



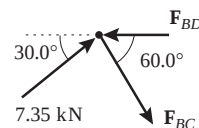
At Pin A:

$$\sum F_y = -7.35 \text{ kN} + 11.0 \text{ kN} - F_{AB} \sin 30.0^\circ = 0$$

$$F_{AB} = 7.35 \text{ kN (compression)}$$

$$\sum F_x = F_{AC} - (7.35 \text{ kN}) \cos 30.0^\circ = 0$$

$$F_{AC} = 6.37 \text{ kN (tension)}$$



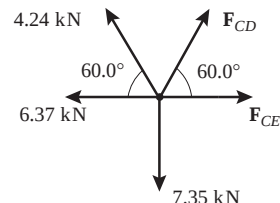
At Pin B:

$$\sum F_y = -(7.35 \text{ kN}) \sin 30.0^\circ - F_{BC} \sin 60.0^\circ = 0$$

$$F_{BC} = 4.24 \text{ kN (tension)}$$

$$\sum F_x = (7.35 \text{ kN}) \cos 30.0^\circ + (4.24 \text{ kN}) \cos 60.0^\circ - F_{BD} = 0$$

$$F_{BD} = 8.49 \text{ kN (compression)}$$



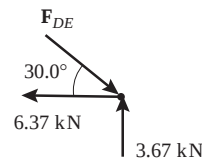
At Pin C:

$$\sum F_y = (4.24 \text{ kN}) \sin 60.0^\circ + F_{CD} \sin 60.0^\circ - 7.35 \text{ kN} = 0$$

$$F_{CD} = 4.24 \text{ kN (tension)}$$

$$\sum F_x = -6.37 \text{ kN} - (4.24 \text{ kN}) \cos 60.0^\circ + (4.24 \text{ kN}) \cos 60.0^\circ + F_{CE} = 0$$

$$F_{CE} = 6.37 \text{ kN (tension)}$$



At Pin E:

$$\sum F_y = -F_{DE} \sin 30.0^\circ + 3.67 \text{ kN} = 0$$

$$F_{DE} = 7.35 \text{ kN (compression)}$$

or $\sum F_x = -6.37 \text{ kN} - F_{DE} \cos 30.0^\circ = 0$
which gives $F_{DE} = 7.35 \text{ kN}$ as before.

FIG. P12.69

- P12.70** (1) $ph = I\omega$
 (2) $p = Mv_{\text{CM}}$

If the ball rolls without slipping, $R\omega = v_{\text{CM}}$

$$\text{So, } h = \frac{I\omega}{p} = \frac{I\omega}{Mv_{\text{CM}}} = \frac{I}{MR} = \boxed{\frac{2}{5}R}$$

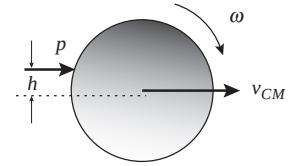


FIG. P12.70

- P12.71** (a) If the acceleration is a , we have $P_x = ma$ and $P_y + n - F_g = 0$. Taking the origin at the center of gravity, the torque equation gives

$$P_y(L - d) + P_x h - nd = 0.$$

Solving these equations, we find

$$P_y = \left[\frac{F_g}{L} \left(d - \frac{ah}{g} \right) \right].$$

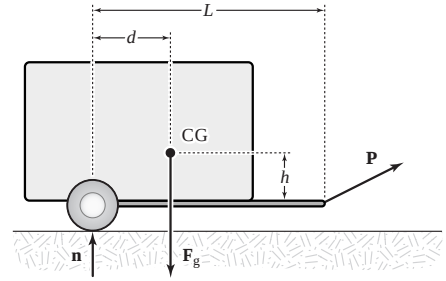


FIG. P12.71

- (b) If $P_y = 0$, then $d = \frac{ah}{g} = \frac{(2.00 \text{ m/s}^2)(1.50 \text{ m})}{9.80 \text{ m/s}^2} = \boxed{0.306 \text{ m}}$.

- (c) Using the given data, $P_x = -306 \text{ N}$ and $P_y = 553 \text{ N}$.

$$\text{Thus, } \mathbf{P} = (-306\hat{\mathbf{i}} + 553\hat{\mathbf{j}}) \text{ N}.$$

- *P12.72** When the cyclist is on the point of tipping over forward, the normal force on the rear wheel is zero. Parallel to the plane we have $f_1 - mg \sin \theta = ma$. Perpendicular to the plane, $n_1 - mg \cos \theta = 0$. Torque about the center of mass:

$$mg(0) - f_1(1.05 \text{ m}) + n_1(0.65 \text{ m}) = 0.$$

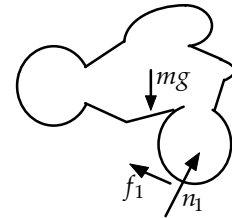


FIG. P12.72

Combining by substitution,

$$ma = f_1 - mg \sin \theta = \frac{n_1 0.65 \text{ m}}{1.05 \text{ m}} - mg \sin \theta = mg \cos \theta \frac{0.65 \text{ m}}{1.05 \text{ m}} - mg \sin \theta$$

$$a = g \left(\cos 20^\circ \frac{0.65}{1.05} - \sin 20^\circ \right) = \boxed{2.35 \text{ m/s}^2}$$

- *P12.73** When the car is on the point of rolling over, the normal force on its inside wheels is zero.

$$\sum F_y = ma_y: \quad n - mg = 0$$

$$\sum F_x = ma_x: \quad f = \frac{mv^2}{R}$$

Take torque about the center of mass: $fh - n\frac{d}{2} = 0$.

$$\text{Then by substitution } \frac{mv_{\text{max}}^2}{R} h - \frac{mgd}{2} = 0 \quad v_{\text{max}} = \sqrt{\frac{gdR}{2h}}$$

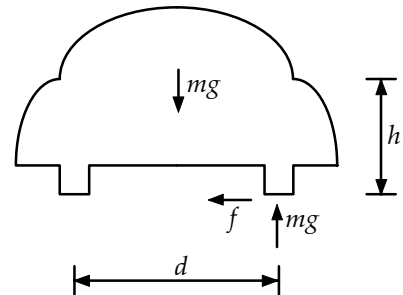


FIG. P12.73

A wider wheelbase (larger d) and a lower center of mass (smaller h) will reduce the risk of rollover.

ANSWERS TO EVEN PROBLEMS

- P12.2** $F_y + R_y - F_g = 0; F_x - R_x = 0;$
 $F_y \ell \cos \theta - F_g \left(\frac{\ell}{2} \right) \cos \theta - F_x \ell \sin \theta = 0$
- P12.4** see the solution
- P12.6** 0.750 m
- P12.8** (2.54 m, 4.75 m)
- P12.10** (a) 9.00 g; (b) 52.5 g; (c) 49.0 g
- P12.12** (a) 392 N; (b) $(339\hat{i} + 0\hat{j})$ N
- P12.14** (a) $f = \left[\frac{m_1 g}{2} + \frac{m_2 g x}{L} \right] \cot \theta;$
 $n_g = (m_1 + m_2)g; (b) \mu = \frac{\left(\frac{m_1}{2} + \frac{m_2 d}{L} \right) \cot \theta}{m_1 + m_2}$
- P12.16** see the solution; 0.643 m
- P12.18** 36.7 N to the left; 31.2 N to the right
- P12.20** (a) 35.5 kN; (b) 11.5 kN to the right;
(c) 4.19 kN down
- P12.22** (a) 859 N; (b) 104 kN at 36.9° above the horizontal to the left
- P12.24** $\frac{3L}{4}$
- P12.26** (a) see the solution; (b) θ decreases;
(c) R decreases
- P12.28** (a) 73.6 kN; (b) 2.50 mm
- P12.30** ~ 1 cm
- P12.32** 9.85×10^{-5}
- P12.34** 0.029 3 mm
- P12.36** (a) -0.0538 m^3 ; (b) $1.09 \times 10^3 \text{ kg/m}^3$;
(c) Yes, in most practical circumstances
- P12.38** (a) 53.1° ; (b) 1.04 kN; (c) 0.126 m, 51.2° ;
(d) 1.07 kN; (e) 0.129 m, 51.1° ; (f) 51.1°
- P12.40** (a) 0.400 mm; (b) 40.0 kN; (c) 2.00 mm;
(d) 2.40 mm; (e) 48.0 kN
- P12.42** at A: $Mg \frac{\sin \beta}{\sin(\alpha + \beta)}$; at B: $Mg \frac{\sin \alpha}{\sin(\alpha + \beta)}$
- P12.44** (a) 160 N to the right;
(b) 13.2 N to the right; (c) 292 N up;
(d) 192 N
- P12.46** 1.46 kN; $(1.33\hat{i} + 2.58\hat{j})$ kN
- P12.48** 0.789
- P12.50** $T = 1.68 \text{ kN}; R = 2.34 \text{ kN}; \theta = 21.2^\circ$
- P12.52** (a) see the solution; (b) 60.0°
- P12.54** (a) 120 N; (b) 0.300; (c) 103 N at 31.0° above the horizontal to the right
- P12.56** (a), (b) see the solution;
(c) $C_{AB} = 732 \text{ N}; T_{AC} = 634 \text{ N}; C_{BC} = 897 \text{ N}$
- P12.58** (a) (9.09 m, 10.9 m); (b) (10.0 m, 10.9 m);
(c) 0.114 m/s to the right
- P12.60** $\frac{3}{8} F_g$
- P12.62** (a) $P_1 = 1.67 \text{ N}; P_2 = 3.33 \text{ N}; P_3 = 1.67 \text{ N};$
(b) 2.36 N
- P12.64** 4.90 cm
- P12.66** 16.7 cm above the center of the bottom
- P12.68** $C_{AB} = 10.4 \text{ kN}; T_{AC} = 7.94 \text{ kN};$
 $T_{BC} = 10.4 \text{ kN}; C_{BD} = 15.9 \text{ kN};$
 $C_{DE} = 10.4 \text{ kN}; T_{DC} = 10.4 \text{ kN};$
 $T_{EC} = 7.94 \text{ kN}$
- P12.70** $\frac{2}{5} R$
- P12.72** 2.35 m/s^2